Causal Frameworks

Applied Econometrics Resources

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Potential Outcomes

We want to find the effect of a binary treatment D on an outcome Y.

Imagine two hypothetical worlds that are identical, except for treatment D:

- World 0: individual i does not get the treatment. $D_i = 0$
- World 1: individual i gets the treatment $D_i = 1$.

Lets imagine their outcome values in each hypothetical world are as follows:

- World 0 no treatment: $Y_i(0)$.
- World 1 with treatment $Y_i(1)$.

$Y_i(0)$ and $Y_i(1)$ are called the **potential outcomes** of the hypothetical worlds.

Causal Effect

The two hypothetical worlds are identical except for treatment D.

Thus, any difference in their outcome values $Y_i(0)$ and $Y_i(1)$ must be **caused** by the difference in treatment.

$$\tau_i = Y_i(1) - Y_i(0)$$

Issue: there isn't actually two hypothetical worlds.

- In the real world, individual i either has the treatment $D_i = 1$, or does not have the treatment $D_i = 0$.
- Thus, we always are missing one of $Y_i(1)$ or $Y_i(0)$. The one we observe is the real Y_i value. The missing one is called a **counterfactual**.

Fundamental Problem of Causal Inference: the causal effect requires us to know both potential outcomes, but we never see both.

Causal Estimands

τ_i is the individual causal effect for individual i. We are often interested in group causal effects:

1. **Average Treatment Effect (ATE)**:

   $$\mathbb{E}[\tau_i] = \mathbb{E}[Y_i(1) - Y_i(0)]$$

2. **Average Treatment Effect on the Treated (ATT)**: average for only those who receive treatment.

   $$\mathbb{E}[\tau_i | D_i = 1] = \mathbb{E}[Y_i(1) - Y_i(0) | D_i = 1]$$

3. **Conditional/Local Average Treatment Effect (CATE/LATE)**: average based on some condition of X.

   $$\mathbb{E}[\tau_i | X_i = 1] = \mathbb{E}[Y_i(1) - Y_i(0) | X_i = 1]$$

Correlation not Causation

Correlation ρ is when we don't consider potential outcomes, but simply compare those who are treated to those who are not treated. Mathematically:

$$\begin{aligned}
\rho_{D,Y} &= \mathbb{E}[Y_i|D_i = 1] - \mathbb{E}[Y_i|D_i = 0] \\
&= \mathbb{E}[Y_i(1)|D_i = 1] - \mathbb{E}[Y_i(0)|D_i = 0] \\
&= \mathbb{E}[Y_i(1)|D_i = 1] - \mathbb{E}[Y_i(0)|D_i = 0] + \mathbb{E}[Y_i(1)|D_i = 1] - \mathbb{E}[Y_i(1)|D_i = 1] \\
&= \underbrace{\mathbb{E}[Y_i(1)|D_i = 1] - \mathbb{E}[Y_i(1)|D_i = 1]}_{\text{ATT}} + \underbrace{\mathbb{E}[Y_i(0)|D_i = 1] - \mathbb{E}[Y_i(0)|D_i = 0]}_{\text{Selection Bias}}
\end{aligned}$$

We can see that correrlation equals ATT + Selection Bias.

- Read: Correlation does not equal causation (ATT) unless selection bias equals 0.

Selection Bias

Recall correlation $=$ ATT $+$ Selection bias. What is selection bias?

$$\text{Selection Bias} = \mathbb{E}[Y_i(0)|D_i = 1] - \mathbb{E}[Y_i(0)|D_i = 0]$$

What does this mean? The two parts:

1. Hypothetical $Y_i(0)$ for those who did receive the treatment $D_i = 0$.
2. Hypothetical $Y_i(0)$ for those who did not receive the treatment $D_i = 0$.

Recall $Y_i(0)$ is hypothetical Y_i without treatment. What this means is:

- Selection bias is the difference **before** (without) **treatment** between the treatment group $D_i = 1$ and treated group $D_i = 0$.

Confounders

What causes selection bias?

- Read: what causes pre-existing differences between treatment $D_i = 1$ and control $D_i = 0$?

Confounders: a variable X that meets two conditions:

1. X causes D.

   Read: individuals with different values of X have different likelihoods of being assigned to $D_i = 1$ or $D_i = 0$

2. X is correlated with Y.

   Read: X is associated with the values of Y.

Thus, since X is associated with Y, and different values of X have different likelihoods of being assigned to D, then D will have different Y values if X is a confounder.

Exogeneity (1)

Recall OLS is only unbiased if strict exogeneity is met $\text{Cov}(X, \varepsilon) = 0$.

Why would X and ε be correlated? Imagine this "true" regression with independent variables X and Z that are correlated:

$$Y_i = \beta_0 + \beta_1 X_i + \beta_2 Z_i + \varepsilon_i$$

Now imagine we forget to include Z in our regression:

$$Y_i = \gamma_0 + \gamma_1 X_i + u_i$$

Now the error term u_i contains Z_i (since error term is everything that explains Y that is not in our regression).

But X and Z are correlated. Since Z is a part of u_i, then X and u_i are correlated, violating exogeneity. The estimate of γ_1 would be biased.

Controlling

Here is our regression of treatment D on outcome Y:

$$Y_i = \alpha + \tau D_i + \varepsilon_i$$

Confounders X is correlated with D by definition.

Thus, if we do not include confounder X in our regression, it will be contained in ε_i, violating exogeneity, and thus creating a biased estimate.

- This matches with the idea of selection bias: not accounting for X means correlation does not equal causation.

Easy solution: add all confounders into regression to meet exogeneity and acheive unbiased estimates:

$$Y_i = \alpha + \tau D_i + \beta X_i + \varepsilon_i$$

Unobservable Confounders

Sounds simple, just control for all confounders X? Then we can get unbiased causal estimates?

Unfortunately, we often run into issues:

1. Confounder X might be impossible/difficult to measure (ex. happiness).
2. We might not have data on X.
3. We might not know all confounders X. There is no magic test to tell us what is a confounder.

Even missing just one X will bias our results.

Thus, the field of **causal inference** exists to find creative ways to get causal estimates.

Random Experiments

The "best" way to isolate causal effects is with random assignment of treatment D_i.

- Our issue is that confounders X cause who gets treatment D.
- If we randomly assign people to treatment D, then it is randomness causing D, and no longer X.

Thus, random assignment solves confounder problems and ensures exogeneity, allowing us to run a simple regression to find causal effects:

$$Y_i = \alpha + \tau D_i + \varepsilon_i$$

Issues:

1. You can't always run randomised experiments. You need to control D, and that can be costly/impractical.
2. Even if you randomly assign D_i, how do you force everyone to follow their assignment?

Observational Studies

We saw random experiments can be difficult to run. Thus, the field of causal inference has designed different techniques to estimate causal effects.

1. **Difference-in-Differences** (DiD): exploiting variation in treatment adoption over time and between individuals.

   DiD is the most popular method right now, and has seen rapid advancement in techniques in the past few years.

2. **Instrumental Variables** (IV): exploiting instruments to estimate causal effects.

   Becoming less popular due to concerns over assumptions

3. **Regression Discontinuity Design** (RDD): exploiting cut-offs in treatment assignment to estimate causal effects.

   Probably the most "convincing" method, but only applicable to certain scenarios.

All methods require **assumptions**. We should always ask if these assumptions are met.